

Practice Test C: Limits, Continuity, IVT

Name: _____

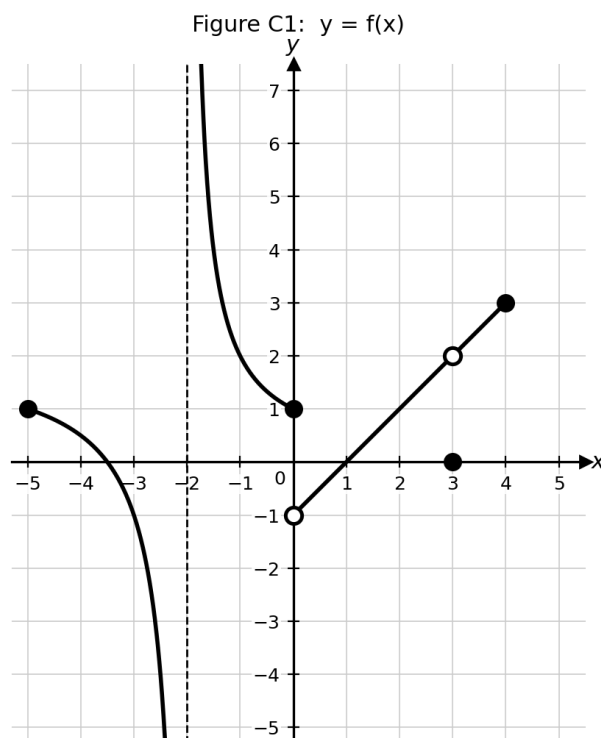
Time: 60 minutes No calculator 100 points

Answer exactly the question asked. Show work on every problem in Parts III–V.

If a limit is infinite, write ∞ or $-\infty$. If a limit does not exist for any other reason, write DNE.

If a function value does not exist, write *undefined*.

Part I. Graph. Questions 1(a)–1(g) refer to Figure C1.



1. Use the figure to find each value.

(a) $\lim_{x \rightarrow -2^-} f(x)$ _____

(b) $\lim_{x \rightarrow -2} f(x)$ _____

(c) $f(-2)$ _____

(d) $\lim_{x \rightarrow 0^-} f(x)$ _____

(e) $\lim_{x \rightarrow 0^+} f(x)$ _____

(f) $f(3)$ _____

(g) $\lim_{x \rightarrow 3} f(x)$ _____

(2 pts each, 14 pts)

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Part II. Table.

2. The table gives values of k . Assume the pattern continues as x approaches 1. **(2 each, 6 pts)**

x	0.9	0.99	0.999	1	1.001	1.01	1.1
$k(x)$	3.729	3.970299	3.997003	undefined	4.003003	4.030301	4.331

(a) $\lim_{x \rightarrow 1^+} k(x)$ _____ (b) $\lim_{x \rightarrow 1} k(x)$ _____ (c) $k(1)$ _____

Part III. Algebra. Show work.

3. Evaluate $\lim_{x \rightarrow -1} \frac{x^2 - 4}{x^2 + x - 6}$, if it exists. **(5 pts)**

4. Evaluate $\lim_{x \rightarrow 12} \frac{\sqrt{x+4} - 2}{x - 4}$, if it exists. **(5 pts)**

5. Evaluate $\lim_{x \rightarrow 1} \frac{x^3 + 2x^2 - 5x + 2}{x^2 + 2x - 3}$, if it exists. **(6 pts)**

6. Let $R(x) = \frac{x - 5}{x^2 - 9}$. Evaluate each limit. **(3 each, 9 pts)**

(a) $\lim_{x \rightarrow 3^-} R(x)$ (b) $\lim_{x \rightarrow 3^+} R(x)$ (c) $\lim_{x \rightarrow 3} R(x)$

7. Let $p(x) = \begin{cases} x^2 - 1 & x < 2 \\ 7 & x = 2 \\ 5 - x & x > 2 \end{cases}$ (2 each, 10 pts)
- (a) $\lim_{x \rightarrow 4} p(x)$ (b) $\lim_{x \rightarrow 2^-} p(x)$ (c) $\lim_{x \rightarrow 2^+} p(x)$ (d) $\lim_{x \rightarrow 2} p(x)$ (e) $p(2)$

8. Let $w(x) = \frac{x^2 - 4x}{x - 4}$. Find $w(4)$, if it is defined. (3 pts)

Part IV. Continuity. For 9 and 10, check each of the three conditions in the definition of continuity at a point and state your conclusion.

9. Using Figure C1, determine whether f is continuous at $x = 3$. (6 pts)

10. Let $h(x) = \begin{cases} 2x + 3 & x < 1 \\ 4 & x = 1 \\ x^2 + 4 & x > 1 \end{cases}$

Determine whether h is continuous at $x = 1$. (6 pts)

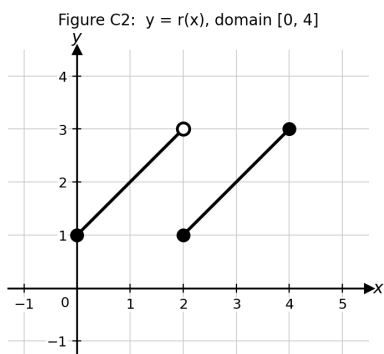
11. Using Figure C1, is f continuous on the closed interval $[0, 2]$? Explain. (4 pts)

12. Let $q(x) = \frac{x+2}{x^2-x-12}$. (3 each, 6 pts)
- (a) Is q continuous on $[-3, 0]$? Explain. (b) Is q continuous on $[-2, 3]$? Explain.

Part V. Intermediate Value Theorem.

13. The function $g(x) = x^3 - 4x^2 + 1$ is continuous on $[0, 3]$. Find $g(3)$. (2 pts)
14. Let $f(x) = x^2 + 2x$ on the interval $[0, 3]$. Does the Intermediate Value Theorem guarantee a number c in $(0, 3)$ with $f(c) = 8$? If it does, find every such c in $(0, 3)$. (6 pts)

15. Figure C2 shows the graph of r on its domain $[0, 4]$. Does the Intermediate Value Theorem guarantee a number c in $[0, 4]$ with $r(c) = 2$? Explain. (4 pts)



16. True or False. Circle one. (4 each, 8 pts)
- (a) **T** **F** If f is continuous on $[0, 4]$, $f(0) = -3$, and $f(4) = 5$, then there is at least one number c in $[0, 4]$ with $f(c) = 0$.
- (b) **T** **F** If $f(1) = 2$, $f(3) = -2$, and f is not continuous on $[1, 3]$, then there is no number c in $(1, 3)$ with $f(c) = 0$.